# IMPLICIT LARGE EDDY SIMULATION OF INCOMPRESSIBLE FLOW

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**Abstract**: In a prepared urban scale model of airflow within a complex geometry area the method of Implicit Large Eddy Simulation (ILES) is used. A staggered grid is used for a discretization of the equations describing the incompressible flow. The use of ILES comes from an application of the high resolution method for convective (advection) terms of the momentum equation. To assess the applicability of the ILES method for this purpose, we are in the stage of testing it on simple and well described cases of turbulent flows. The first example of 3D turbulent flow is the Taylor-Green vortex – originally a simple free flow containing organized vortices with a subsequent transition to turbulence. In this contribution there will be shown several characteristics of the turbulent flow and given their comparison with results described in literature (energy spectra, kinetic energy dissipation rate etc.).

***Key words:*** *Large Eddy Simulation, ILES, MILES, Taylor-Green vortex*

## 1. INTRODUCTION

The Implicit Large Eddy Simulation (ILES) is a relatively new way of simulation of turbulent flows. The main difference between ILES and normal Large Eddy Simulation is usage numerical dissipation of so-called hi resolution schemes (schemes that are at least second order in smooth areas and do not produce unphysical wiggles near discontinuities) instead of explicit subgrid stress models. The fundamentals of ILES lie in the field of compressible computational fluid dynamics with schock and similar phenomena. Many of compressible CFD schemes were applied to ILES, for example flux-corrected transport (FCT), partial parabolic method (PPM) or MPDATA which is aimed for geophysical applications. One of important points in these methods is the flux or slope limiters, which introduce nonlinearity in the computations and prevent the emergence of oscillations near discontinuities.

Our aim is to develop a similar ILES code, but in the framework of traditional incompressible flows on a staggered grid. The staggered grid does not store all velocity components in the same cells and therefore causes additional problems. We developed a 3D version of projection method of Tau (1994), which is an adoption of a method by Bell et al. (1989) to a staggered grid. Detailed description of our method can be found in (Fuka and Brechler, 2008). It belongs to a class of projection (or fractional step) methods that solve first a momentum equation alone for a velocity field that does not follow a continuity equation. In the second step, this velocity field is projected onto a nondivergent field. In this contribution we use a slightly updated version of our method that uses upwinding for corner values in all directions.

As a first test case we computed Taylor-Green vortex flow, which serves as a simple example of free flow with transition to turbulence with a subsequent turbulent decay. This flow was used for this purpose by several other authors, for example Garnier et al. (1999). The comparison is usually done with direct numerical simulation (DNS) data of Brachet (1983) and Brachet (1991). In these articles one can also find the initial and boundary conditions for this flow.

## 2. RESULTS

Even though Taylor-Green vortex has very simple boundary conditions, it shows a complex behaviour. At the beginning the flow is purely two-dimensional, but as flow continues, it forms of vortex sheets, that break into smaller eddies, and after t = 5 the flow becomes turbulent. According to DNS of Brachet (1991) at t=9 is the peak of the kinetic energy dissipation. After this peak the turbulent eddies dissipate in the self-similar energy cascade.

We computed several cases for various slope limiters and various resolutions. The definitions of the slope limiters can be found in (Fuka and Brechler, 2008). we used no molecular diffusion and we compared the results to DNS of Brachet (1991) at the Reynolds number 5000. The main indicators we used for comparison are the time history of the kinetic energy and its dissipation, kinetic energy spectrum and PDFs of pressure and velocity gradients. The results are in Figures 1-5. The k. e. dissipation shows a good agreement with DNS for different limiters. On the other hand kinetic energy spectrum shows an inertial range with a power law different from the Kolmogorov one. The PDFs are consistent with the expectations, with exponential tails for velocity gradients and strong asymmetry for pressure.

## 3. CONLUSIONS

We developed a 3D ILES model for incompressible flows on a staggered grid and we tested it on a simple case of Taylor-Green vortex. We used several criteria for assessment of the performance of the model with success in some and with less success in others.

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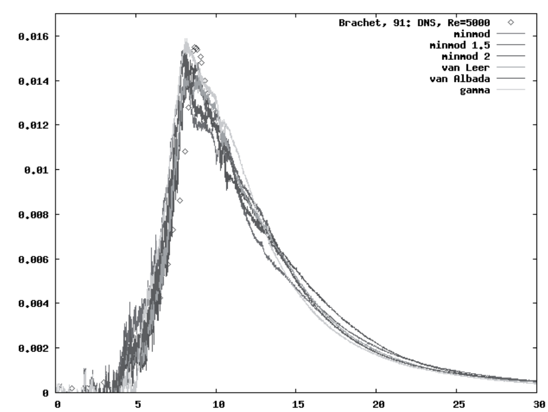


Figure 1. The kinetic energy dissipation for various limiters.

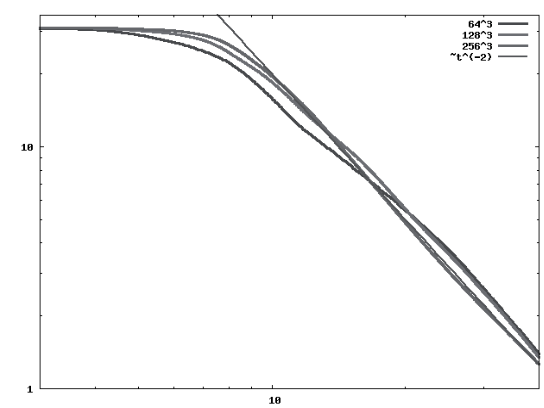


Figure 2. The time history of the kinetic energy for three resolutions for ext. minmod limiter.

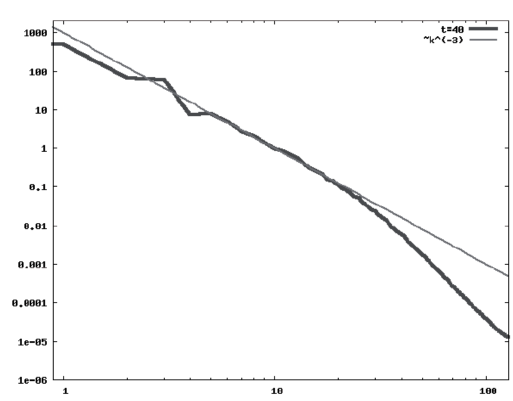


Figure 3. The kinetic energy spectrum in $t = 40$ for ext. minmod limiter.

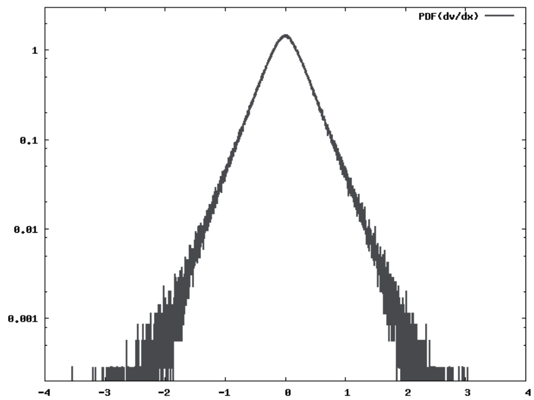


Figure 4. The PDF of the velocity gradient d$v$/d$x$ for the ext. limiter in $t$ = 40.

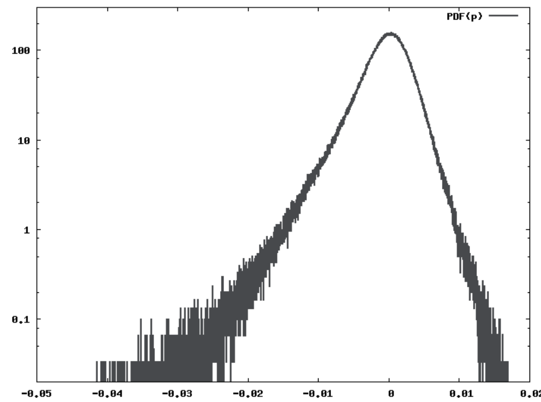


Figure 5. The PDF of the pressure for the ext. limiter in $t$ = 40.